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Spacecraft Technology; Spacecraft Dynamics and Control

Theme

Analytical study of attitude motions of a gyrostat in a resisting medium.

Content

The system under consideration is a gyrostat G consisting of two rigid bodies, A and B. The central inertia ellipsoids of B and G are ellipsoids of revolution with parallel axes; the mass center of B is fixed in A; and B can rotate relative to A about its symmetry axis.

The dynamical equations for this system can be cast into the form

$$I\dot{p}_j = \mathbf{R}\cdot\mathbf{c}_j, \quad j = 1,2,3$$

$$\dot{r} = [J/K(J-K)]\mathbf{c}_3\cdot\mathbf{M} - [\mathbf{R}\cdot\mathbf{c}_3/(J-K)]$$

where I, J, and K are moments of inertia; $\mathbf{c}_1$, $\mathbf{c}_2$, and $\mathbf{c}_3$ are unit vectors, the last of these being parallel to the symmetry axis of G; p_1, p_2, and p_3 are angular velocity components of a reference frame in which $\mathbf{c}_1$, $\mathbf{c}_2$, and $\mathbf{c}_3$ are fixed and in which A rotates with a rate $s = [(I-J)p_3 - Kr]/J$; r is the angular speed of B relative to A; $\mathbf{R}$ is the moment about the mass center of G of all forces applied to A by the resisting medium in which A moves; and $\mathbf{M}$ is the moment about the mass center of B of all forces exerted by A on B.

Taking

$$\mathbf{R} = -\beta I(p_1\mathbf{c}_1 + p_2\mathbf{c}_2) - \gamma(I/J)(Ip_3 - Kr)\mathbf{c}_3$$

where β and γ are constants, one can integrate the equations of motion, both for the case when B is completely free to rotate relative to $A(\mathbf{c}_3\cdot\mathbf{M} = 0)$ and for a rotor driven at constant angular speed ($\dot{r} = 0$); and a perturbation technique can then be used to solve three kinematical equations of the form

$$\dot{n}_1 = n_2p_3 - n_3p_2$$

where $n = \mathbf{n}\cdot\mathbf{c}_2$ and $\mathbf{n}$ is an inertially fixed unit vector which is equal to $\mathbf{c}_3$ at $t = 0$. The results of this integration permit one to find the limiting angle θ^* between $\mathbf{n}$ and the axis of symmetry of G:

$$\theta^* = |a_2(0)|\{\beta^2 + [(K/I)r(0)]^2\}^{-1/2}$$

where $|a_2(0)|$ is the magnitude of the initial transverse angular velocity of A. This formula yields values in good agreement with those resulting from a numerical integration of the kinematical equations, provided ϵ, defined as $\epsilon = a_2(0)/\beta$, is sufficiently small.

Motion of a Symmetric Gyrostat in a Viscous Medium

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Dynamical equations governing the motion of a symmetric gyrostat in a viscous medium are solved in closed form, both for the case of a free rotor and for a rotor driven with constant angular speed. Kinematical equations of the Poisson type are then solved by a perturbation technique to obtain a formula for the limiting angle between the symmetry axis of the gyrostat and the fixed line with which this axis coincides initially; and this formula is tested by means of computer solutions of the kinematical equations.

Introduction

IN the more than sixty years that have elapsed since the publication of Gray's comprehensive work on gyrostats,[1] two major developments bearing directly on this subject have occurred: dual spin satellites have begun to play an important role in the field of space flight, and highly effective computing machines have come into wide use. The first of these suggests that fundamental insights may have more practical significance now than ever before, and the second means that one can employ a powerful, previously unavailable tool to gain such insights. With these ideas in mind, it was decided to seek information relevant to the following fundamental question: How does a dual spin satellite behave in a resisting medium? What follows is an attempt to provide a partial answer to this question by solving a certain problem in dynamics.

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Analysis

The system to be analyzed is a gyrostat G (Fig. 1) consisting of two rigid bodies, A and B. The inertia ellipsoid of A for the mass center A^* of A may have three unequal principal diameters, whereas the inertia ellipsoid E_B of B for the mass center B^* of B is presumed to be an ellipsoid of revolution. Furthermore, B is connected to A in such a way that (1) B^* and the axis of revolution of E_B are fixed in A, but B can rotate relative to A about this axis, and (2) the inertia ellipsoid E_G of the gyrostat G for the mass center G^* of G is an ellipsoid of revolution whose axis is parallel to that of E_B. Torque-free motions of precisely this system were discussed in a previous paper[2] in this Journal, and, for the sake of brevity, information contained in that paper is used wherever possible in the sequel.

Before choosing a mathematical description for the action of the resisting medium on body A, it is convenient to introduce certain inertial and kinematical parameters.

In Fig. 1, $\mathbf{c}_1$, $\mathbf{c}_2$, and $\mathbf{c}_3$ designate mutually perpendicular unit vectors oriented in such a way that $\mathbf{c}_3$ is parallel to the axes of revolution of E_B and E_G, but $\mathbf{c}_1$ and $\mathbf{c}_2$ are not fixed in A or in B. Symbols denoting moments of inertia of interest are defined in terms of these unit vectors and the inertia dyadics $\mathbf{I}^G$ of G for G^* and $\mathbf{I}^B$ of B for B^* by letting

$$I = \mathbf{c}_1 \cdot \mathbf{I}^G \cdot \mathbf{c}_1 = \mathbf{c}_2 \cdot \mathbf{I}^G \cdot \mathbf{c}_2 \tag{1}$$

$$J = \mathbf{c}_3 \cdot \mathbf{I}^G \cdot \mathbf{c}_3 \tag{2}$$

and

$$K = \mathbf{c}_3 \cdot \mathbf{I}^B \cdot \mathbf{c}_3 \tag{3}$$

The angular velocity of B relative to A, denoted by $\mathbf{r}$, can be expressed as

$$\mathbf{r} = r\mathbf{c}_3 \tag{4}$$

where r is a function of time t. Next, a reference frame C, in which $\mathbf{c}_1$, $\mathbf{c}_2$, and $\mathbf{c}_3$ are fixed, is introduced, and it is noted that the angular velocity of A relative to C, denoted by $\mathbf{s}$, is necessarily parallel to $\mathbf{c}_3$ because $\mathbf{c}_3$ is parallel to the axis of revolution of E_B, and this axis is fixed in A. Consequently, $\mathbf{s}$ can be expressed as

$$\mathbf{s} = s\mathbf{c}_3 \tag{5}$$

where s is a function of t. Finally, the angular velocity of C relative to a Newtonian reference frame N is denoted by $\mathbf{p}$ and is expressed in terms of functions p_1, p_2, and p_3 of t as

$$\mathbf{p} = p_1\mathbf{c}_1 + p_2\mathbf{c}_2 + p_3\mathbf{c}_3 \tag{6}$$

The angular velocity $\mathbf{a}$ of A relative to N is thus given by†

$$\mathbf{a} = \mathbf{p} + \mathbf{s} \underset{(6,5)}{=} p_1\mathbf{c}_1 + p_2\mathbf{c}_2 + (p_3 + s)\mathbf{c}_3 \tag{7}$$

The quantity s introduced in Eq. (5) is completely at the analyst's disposal. In Ref. 2, this fact was used to bring the differential equations of motion into a particularly simple form. Precisely the same situation obtains now: If one assigns to s the value

$$s = [(I - J)p_3 - Kr]/J \tag{8}$$

then, proceeding as in Ref. 2, one can use the angular momentum principle to express the dynamical equations governing all motions of the system as

$$I\dot{p}_1 = \mathbf{R} \cdot \mathbf{c}_1 \tag{9}$$

$$I\dot{p}_2 = \mathbf{R} \cdot \mathbf{c}_2 \tag{10}$$

$$I\dot{p}_3 = \mathbf{R} \cdot \mathbf{c}_3 \tag{11}$$

and

$$\dot{r} = [J/K(J - K)]\mathbf{c}_3 \cdot \mathbf{M} - [\mathbf{R} \cdot \mathbf{c}_3/(J - K)] \tag{12}$$

where $\mathbf{R}$ is the moment about G^* of all forces applied to A by the resisting medium in which A moves, and $\mathbf{M}$ is the moment about B^* of all forces exerted by A on B. Furthermore, with s as in Eq. (8), the angular velocity of A in N becomes

$$\mathbf{a} = p_1\mathbf{c}_1 + p_2\mathbf{c}_2 + [(Ip_3 - Kr)/J]\mathbf{c}_3 \tag{13}$$

The action of the resisting medium on A can now be represented in mathematical form by relating $\mathbf{R}$ to $\mathbf{a}$. For example, a description of an essentially viscous resistance is obtained by taking

$$\mathbf{R} = -\beta I(p_1\mathbf{c}_1 + p_2\mathbf{c}_2) - \gamma(I/J)(Ip_3 - Kr)\mathbf{c}_3 \tag{14}$$

where β and γ are non-negative constants which characterize "transverse" and "axial" damping, respectively. For this choice of $\mathbf{R}$, Eqs. (9–12) become

$$\dot{p}_1 = -\beta p_1 \tag{15}$$

$$\dot{p}_2 = -\beta p_2 \tag{16}$$

$$\dot{p}_3 = -\gamma(Ip_3 - Kr)/J \tag{17}$$

† Numbers beneath signs of equality are intended to direct attention to equations numbered correspondingly.

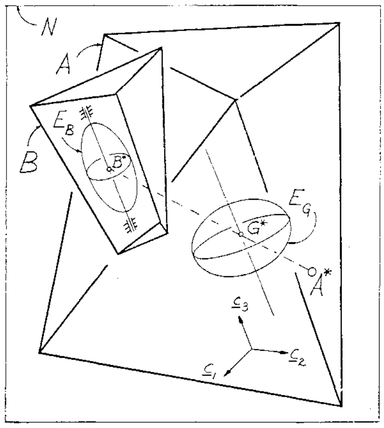

Fig. 1 Gyrostat.

and

$$\dot{r} = \frac{J}{K(J - K)}\mathbf{c}_3 \cdot \mathbf{M} + \gamma\frac{I(Ip_3 - Kr)}{J(J - K)} \tag{18}$$

Eqs. (15) and (16) yield p_1 and p_2 immediately,

$$p_1 = p_1(0)e^{-\beta t}, \quad p_2 = p_2(0)e^{-\beta t} \tag{19}$$

where $p_1(0)$ and $p_2(0)$ denote the initial values of p_1 and p_2. As for p_3 and r, both can be found in two situations of interest. First, if B is driven by a motor in such a way that the angular speed of B relative to A remains constant, and thus equal to its initial value, $r(0)$, then the general solution of Eq. (17) is

$$p_3 = [p_3(0) - (K/I)r(0)]e^{-\gamma(I/J)t} + (K/I)r(0) \tag{20}$$

Second, if B is completely free to rotate relative to A, that is, if $\mathbf{c}_3 \cdot \mathbf{M} = 0$, then

$$\dot{r} \underset{(18)}{=} \gamma I(Ip_3 - Kr)/J(J - K) \tag{21}$$

so that, after eliminating $\gamma(Ip_3 - Kr)/J$ by reference to Eq. (17), one can write

$$\dot{r} = [I/(K - J)]\dot{p}_3 \tag{22}$$

from which it follows that

$$r = [I/(J - K)][p_3(0) - p_3] + r(0) \tag{23}$$

Elimination of r from Eq. (17) then gives

$$\dot{p}_3 + \gamma[I/(J - K)]p_3 = \gamma(K/J)\{r(0) + [I/(J - K)]p_3(0)\} \tag{24}$$

and the general solution of this equation is

$$p_3 = \frac{J - K}{J}\left\{\left[p_3(0) - \frac{K}{I}r(0)\right]e^{-\gamma[I/(J-K)]t} + \frac{K}{I}\left[r(0) + \frac{I}{J - K}p_3(0)\right]\right\} \tag{25}$$

The integration of the dynamical equations has now been completed. However, knowledge of p_1, p_2, p_3 and r as functions of t does not, per se, permit one to give a complete description of the motion of the gyrostat. To be able to give such a description, one must integrate a set of differential equations governing suitable geometric variables. This will be done presently. First, however, it is worth noting that the quantities $p_1(0)$, $p_2(0)$, and $p_3(0)$, which appear in Eqs. (19), (20), and (25), can all be related to the initial angular velocity of body A; that is, if a_i is defined as

$$a_i = \mathbf{a} \cdot \mathbf{c}_i \tag{26}$$

then, from Eq. (13),

$$a_1(0) = p_1(0), \quad a_2(0) = p_2(0) \tag{27}$$

Table 1 Expressions for L, M, and δ

	L	M	δ
Constant speed rotor	J/I	0	$\gamma I/J$
Free rotor	$(J-K)/I$	K/I	$\gamma I(J-K)$

Table 2 Numerical values of L, M, and η

	L	M	η
Constant speed rotor	3/2	0	2/3
Free rotor	45/32	3/32	32/45

and

$$a_3(0) = [Ip_3(0) - Kr(0)]/J \tag{28}$$

so that

$$p_3(0) = [Ja_3(0) + Kr(0)]/I \tag{29}$$
(28)

Eqs. (19) can thus be replaced with

$$p_1 = a_1(0)e^{-\beta t}, \quad p_2 = a_2(0)e^{-\beta t} \tag{30}$$

and, for the constant speed rotor,

$$p_3 = (J/I)a_3(0)e^{-\gamma(I/J)t} + (K/I)r(0) \tag{31}$$
(20,29)

whereas, for the free rotor,

$$p_3 = [(J-K)/I]a_3(0)e^{-\gamma[I/(J-K)]t} + (K/I)[r(0) + a_3(0)] \tag{32}$$
(25,29)

Two further observations simplify the work that follows. The first is that the orientation of the unit vectors $\mathbf{c}_1$ and $\mathbf{c}_2$ at $t = 0$ can always be chosen such that, at this instant, $\mathbf{c}_1$ is perpendicular to $\mathbf{a}$, in which case $a_1(0)$ is equal to zero and Eqs. (30) become

$$p_1 = 0, \quad p_2 = a_2(0)e^{-\beta t} \tag{33}$$

The second observation is that Eqs. (31) and (32) are both contained in

$$p_3 = a_3(0)(Le^{-\delta t} + M) + r(0)(K/I) \tag{34}$$

where L, M, and δ are constants which have the values shown in Table 1.

Substituting from Eqs. (33) and (34) into Eq. (13), and setting r equal to $r(0)$ when dealing with the constant speed rotor, or using Eqs. (23) and (31) for the free rotor, one finds that

$$\mathbf{a} = a_2(0)e^{-\beta t}\mathbf{c}_2 + a_3(0)e^{-\delta t}\mathbf{c}_3 \tag{35}$$

which shows that, as the reader may have anticipated, $\mathbf{a}$ approaches zero as t approaches infinity. In other words, body A eventually comes to rest. What one should like to discover is the limiting orientation of A in N or, at least, the rest angle θ^*, that is, the limiting value of the angle θ between the axis of revolution of E_B and that line fixed in N with which this axis coincides initially.

To determine θ, let $\mathbf{n}$ be a unit vector fixed in N and equal to $\mathbf{c}_3$ at $t = 0$. Then

$$\theta = \cos^{-1}(\mathbf{n}\cdot\mathbf{c}_3) \tag{36}$$

or, if n_1, n_2, and n_3 are three scalars such that

$$\mathbf{n} = n_1\mathbf{c}_1 + n_2\mathbf{c}_2 + n_3\mathbf{c}_3 \tag{37}$$

then

$$\theta = \cos^{-1}n_3 \tag{38}$$

Next, let $\mathbf{n}^N$ and $\mathbf{n}^C$ denote the first time-derivatives of $\mathbf{n}$ in N and in C, respectively. C, it will be recalled, is a reference frame in which $\mathbf{c}_1$, $\mathbf{c}_2$, and $\mathbf{c}_3$ are fixed, and the angular velocity of C relative to N is denoted by p. Hence $\mathbf{n}^N$ and $\mathbf{n}^C$ are related to each other as follows:

$$\mathbf{n}^N = \mathbf{n}^C + \mathbf{p}\times\mathbf{n} \tag{39}$$

where

$$\mathbf{n}^N = 0 \tag{40}$$

because $\mathbf{n}$ is fixed in $\mathbf{R}$ by construction; $\mathbf{n}^C$ can be expressed as

$$\mathbf{n}^C = \dot{n}_1\mathbf{c}_1 + \dot{n}_2\mathbf{c}_2 + \dot{n}_3\mathbf{c}_3 \tag{41}$$

where dots denote differentiation with respect to t; and $\mathbf{p}\times\mathbf{n}$ can be evaluated by reference to Eqs. (6) and (37). Consequently, Eq. (39) leads to three kinematical equations of the Poisson type, namely

$$\dot{n}_1 = n_2p_3 - n_3p_2 \tag{42}$$

$$\dot{n}_2 = n_3p_1 - n_1p_3 \tag{43}$$

$$\dot{n}_3 = n_1p_2 - n_2p_1 \tag{44}$$

and, once these have been integrated subject to the initial conditions

$$n_1(0) = n_2(0) = 0, \quad n_3(0) = 1 \tag{45}$$

which guarantee that $\mathbf{n} = \mathbf{c}$, at $t = 0$, θ can be found by using Eq. (38). Unfortunately, when p_1, p_2, and p_3 are functions of t [see Eqs. (33) and (34)], this integration generally cannot be accomplished in closed form. (The method of solution described in Ref. 3 cannot be used here because, as may be verified by substitution from Eq. (10.50) of Ref. 3 into the last of Eqs. (10.43), this method yields correct results only when the matrices U and W' commute, which is not the case here.) However, one may expect to obtain a practically useful approximate solution of Eqs. (42–44) by restricting oneself to situations in which the initial angular velocity of A is "small." Analytically, this can be accomplished by introducing two constants, ϵ and μ, as

$$\epsilon = a_2(0)/\beta \tag{46}$$

and

$$\mu = a_3(0)/a_2(0) \tag{47}$$

respectively, and then dropping higher powers of ϵ in comparison with lower powers when solving the equations obtained by substituting from Eqs. (33, 34, 46, and 47) into Eqs. (42–44), these equations being

$$\dot{n}_1 = n_2[\mu\epsilon\beta(Le^{-\delta t} + M) + r(0)(K/I)] - n_3\epsilon\beta e^{-\beta t} \tag{48}$$

$$\dot{n}_2 = -n_1[\mu\epsilon\beta(Le^{-\delta t} + M) + r(0)(K/I)] \tag{49}$$

$$\dot{n}_3 = n_1\epsilon\beta e^{-\beta t} \tag{50}$$

The solution of Eqs. (48–50) is simplified somewhat by re-expressing the equations as

$$n_1' = n_2[\mu\epsilon(Le^{-\eta t} + M) + P] - n_3\epsilon e^{-\tau} \tag{51}$$

$$n_2' = -n_1[\mu\epsilon(Le^{-\eta\tau} + M) + P] \tag{52}$$

$$n_3' = n_1\epsilon e^{-\tau} \tag{53}$$

where τ, η, and P are dimensionless quantities defined as

$$\tau = \beta t \tag{54}$$

$$\eta = \delta/\beta \tag{55}$$

and

$$P = r(0)K/\beta I \tag{56}$$

respectively, and where primes denote differentiation with respect to τ.

A solution of Eqs. (51–53) is now sought in the form

$$n_i = \sum_{j=0}^{\infty} \epsilon^j n_{ij}, \quad i = 1,2,3 \tag{57}$$

and, to satisfy Eqs. (45), the following initial conditions are

Table 3 Rest angle, deg

	$P = -1$			$P = 0$			$P = 1$		
ϵ	Const. speed rotor	Free rotor	Eq. (70)	Const. speed rotor	Free rotor	Eq. (70)	Const. speed rotor	Free rotor	Eq. (70)
0.01	0.42	0.42	0.41	0.58	0.58	0.57	0.42	0.42	0.41
0.05	2.07	2.07	2.03	2.87	2.87	2.87	1.99	1.99	2.03
0.10	4.21	4.21	4.05	5.72	5.72	5.73	3.89	3.89	4.05
0.20	8.72	8.74	8.10	11.38	11.37	11.46	7.44	7.43	8.10
0.50	23.87	23.97	20.37	27.39	27.29	28.65	16.02	15.98	20.37
	1	2	3	4	5	6	7	8	9

imposed:

$$n_{10}(0) = n_{20}(0) = 0, \quad n_{30}(0) = 1 \tag{58}$$

and

$$n_{ij}(0) = 0, \quad i = 1,2,3; \; j = 1,2,\ldots,\infty \tag{59}$$

Substituting from Eq. (57) into Eqs. (51–53), and setting the coefficient of each power of ϵ in each of the resulting equations equal to zero, one is led to

$$n_{10}' = Pn_{20}, \quad n_{20}' = -Pn_{10}, \quad n_{30}' = 0 \tag{60}$$

and, for $j = 1,2,\ldots,$

$$n_{1j}' = Pn_{2j} + \mu(Le^{-\eta\tau} + M)n_{2j-1} - e^{-\tau}n_{3j-1} \tag{61}$$

$$n_{2j}' = -Pn_{1j} - \mu(Le^{-\eta\tau} + M)n_{1j-1} \tag{62}$$

$$n_{3j}' = e^{-\tau}n_{1j-1} \tag{63}$$

The solution of Eqs. (60) that satisfies Eqs. (58) is

$$n_{10} = n_{20} = 0, \quad n_{30} = 1 \tag{64}$$

and, using these results in conjunction with Eqs.(59) to solve Eqs. (61–63), first with $j = 1$ and then with $j = 2$, one obtains

$$n_{31} = 0 \tag{65}$$

and

$$n_{32} = -[1/2(1 + P^2)](1 + e^{-2\tau} - 2e^{-\tau}\cos P\tau) \tag{66}$$

Taking $i = 3$ in Eqs. (57), and dropping all terms involving powers of ϵ above the second, one thus finds that n_3 is given, approximately, by

$$n_3 \approx 1 - [\epsilon^2/2(1 + P^2)](1 + e^{-2\tau} - 2e^{-\tau}\cos P\tau) \tag{67}$$

from which it follows that

$$\lim_{\tau\to\infty} n_3 \approx 1 - [\epsilon^2/2(1 + P^2)] \tag{68}$$

The rest angle $\theta^*$is thus given by

$$\theta^* \underset{(38)}{=} \lim_{t\to\infty} \theta \approx |\epsilon|/(1 + P^2)^{1/2} \tag{69}$$

or, after elimination of ϵ and P by reference to Eqs. (41) and (56), by

$$\theta^* \approx \frac{|a_2(0)|}{\{\beta^2 + [(K/I)r(0)]^2\}^{1/2}} \tag{70}$$

Discussion

The validity of Eq. (70) can be tested by comparing values of θ^* predicted by this equation with corresponding values obtained by using Eq. (38) together with the results of a computer integration of Eqs. (51–53). In order to perform such calculations, one must assign numerical values to ϵ, μ, η, L, M, and P. Physically meaningful values of these parameters can be generated by considering a representative example. Suppose, for instance, that A is a uniform, right-circular, cylindrical shell of height h, with inner and outer radii equal to $h/2$ and h, respectively, and that B is a uniform, right-circular cylinder, made of the same material as A and having a height h and radius $h/2$. Then $J/I = \frac{3}{2}$ and $K/I = \frac{3}{32}$. Suppose further that the damping constants β and γ, introduced in Eq. (14), are equal to each other, and that the angular velocity vector of A is initially inclined at forty-five degrees to the common axis of A and B, so that $a_1(0)$ is equal to $a_3(0)$. Then, from Eq. (47), $\mu = 1$, and L, M, and η, found by reference to Table 1 and Eq. (55), have the values shown in Table 2. Next, as may be seen by reference to Eq. (56), the magnitude and the sign of P may be chosen at will by simply assigning an appropriate value to the initial rotor speed $r(0)$. The values -1, 0, and 1 are thus representative. Finally [see Eq. (46)], ϵ furnishes a measure of the relative magnitudes of the constant β and the initial angular velocity of A. Hence, by arbitrarily using the values 0.01, 0.05, 0.10, 0.20, and 0.50 for ϵ, one acquires a rather broad basis for an assessment of the validity of Eq. (70).

The numbers recorded in columns 1, 2, 4, 5, 7, and 8 of Table 3 were obtained by carrying computer integrations to a point at which changes were no longer occurring in the third decimal place. Hence, these entries represent "exact" values of θ^*. Columns 3, 6, and 9 contain corresponding values of θ^* as given by Eq. (70).

Eq. (70) applies both when the rotor is free and when the rotor speed is kept constant. Hence, if this equation is to be correct, there can be, at most, a small difference between any two entries in the same row of columns 1 and 2, 4 and 5, or 7 and 8. This is seen to be the case, and the differences in question grow with increasing ϵ. This means that the validity of Eq. (70) can be expected to suffer as ϵ increases, which is not surprising. Similarly, the agreement between corresponding entries in columns 1 (or 2) and 3, 4 (or 5) and 6, and 7 (or 8) and 9 is better for smaller values of ϵ than it is for larger values. What is both gratifying and, perhaps, surprising is that these differences remain smaller than one degree even when the rest angle becomes as large as eleven degrees. Eq. (70) thus appears to have a rather wide range of applicability.

If $r(0)$ is set equal to zero, the constant rotor speed problem reduces to that of a single rigid body in a viscous medium. A comparison of columns 4 and 6 shows that Eq. (70) yields particularly good results in this case. Perhaps this is due to the fact that, as may be easily verified, Eq. (70) furnishes an exact, rather than merely an approximate, expression for the rest angle of a rigid body if $a_3(0)$ is equal to zero.

In conclusion, it may be worth noting that the absence of J and $a_3(0)$ from Eq. (70) suggests that these quantities have a relatively small effect on θ^*, and this has a clear-cut practical implication: Stabilization, in the sense of reduction of the rest angle, can be accomplished more effectively by increasing the initial rotor angular momentum, $Kr(0)$, than by increasing the initial spin angular momentum, $Ja_3(0)$, of the gyrostat.

References

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